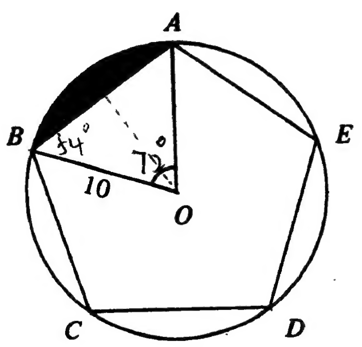
92

Solutions	Marks	Remarks
1. (a) $\frac{\pi}{6}$ (radian) (or 0.167π or 0.17 ×)	1A	
(b) $x = 150°$ $(\frac{5\pi}{6}, 2.62)$	1A	
(c) $\cos A$	1A	
	3	
2. (a) $p + q$	1A	
(b) -2	1A	
(c) $\sqrt{3} - \sqrt{2}$	1A	
	3	
3. (a) $y \geq \frac{1}{2}$	1A	Withhold 1 mk if '=' omitted
$2x - y \geq 2$	1A	
$3x + 5y \leq 30$	1A	
(b) 16	1A	
	4	
4. (a) (i) $x^2 - 2x = x(x - 2)$	1A	$x^2-2x\neq 0$ $x(x-2)\neq 0$
(ii) $x^2 - 6x + 8 = (x - 2)(x - 4)$	1A	
(b) $\frac{1}{x^2 - 2x} + \frac{1}{x^2 - 6x + 8} = \frac{1}{x(x - 2)} + \frac{1}{(x - 2)(x - 4)}$		
$= \frac{(x - 4) + x}{x(x - 2)(x - 4)}$	1M ?	
$= \frac{2x - 4}{x(x - 2)(x - 4)}$ collect like terms	1A	
$= \frac{2}{x(x - 4)}$ $\left(= \frac{2}{x^2 - 4x} \right)$	1A	
	5	
5. (a) Slope of $L_2 = \frac{1}{2}$	1A	
Slope of $L_1 = -2$		
Equation of L_1 : $y - 5 = -2(x - 10)$	1M	Pt-slope form
i.e. $2x + y - 25 = 0$ (or $y = \ldots$)	1A	
(b) Solving $\begin{cases} x - 2y + 5 = 0 \\ 4x + 2y - 50 = 0 \end{cases}$		
$5x - 45 = 0$	1M	Eliminating 1 unknown
$x = 9$ (or $y = 7$)	1A	
∴ L_1 and L_2 meet at $(9, 7)$	1A	Accept $x = 9$, $y = 7$
	6	

Solutions	Marks	Remarks
6. For distinct real roots		
$\Delta = (2k)^2 - 4(k+6) > 0$	2M+1A	1A for $(2k)^2 - 4(k+6)$ 2M for $\Delta > 0$ ($\Delta \geq 0$, 1M only)
$4k^2 - 4k - 24 > 0$		
$(k+2)(k-3) > 0$	1A	For $(k+2)(k-3)$
$\therefore \quad k < -2$ or $k > 3$	2A	For ',', ',' '=' withhold 1 mk each $k<-2$ and $k>3$ (0分)
	6	
7. (a) $\angle AOB = \frac{360°}{5} = 72°$ $\left(= \frac{2\pi}{5} \approx 1.26 \text{ radians} \right)$	1A	
72		
Area of $\triangle OAB = \frac{1}{2}(10)(10)\sin 72°$	1M	
$= 47.6$ (47.5528)	1A	Any figure roundable to 47.6
(b) Area of sector $OAB = \frac{1}{5} \cdot \pi 10^2$	1A	
$\frac{1}{2} \times 10^2 \times \left(\frac{72\pi}{180}\right)$		
$= 20\pi$ (62.83)		
Area of shaded part $= 20\pi - 47.55$	1M	
$= 15.3$ (15.2790)	1A	Accept 15.2 ~ 15.3
	6	
8. (a) Total score of the team $= 70(m+n)$	1A	
(b) Total score is also equal to $75m + 62n$.	1A	
$75m + 62n = 70(m+n)$	1M	
$5m = 8n$		
$m : n = 8 : 5$ $\left(= \frac{8}{5} \right)$	1A	
(c) The number of men $= 39 \times \frac{8}{8+5}$	1M	
$= 24$	1A	
	6	

$gm = 11.75$



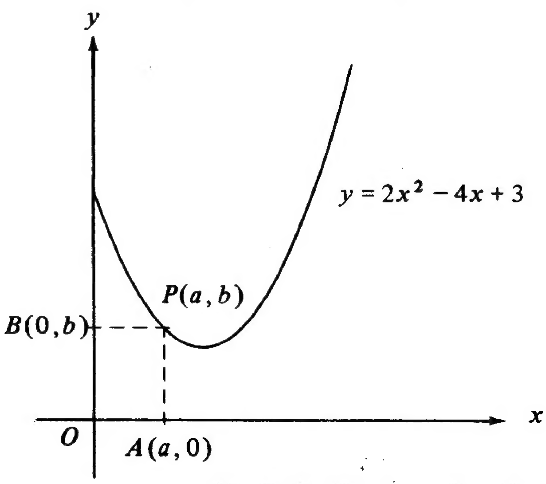
Solutions	Marks	Remarks
9. (a) (i) Area of $OAPB = a \times b$	1A	
$= a(2a^2 - 4a + 3)$	1A	
$= 2a^3 - 4a^2 + 3a$		
(ii) For $OAPB$ to be a square, $a = b$	1M	Equating adjacent sides
$a = 2a^2 - 4a + 3$		
$2a^2 - 5a + 3 = 0$	1A	
$(2a - 3)(a - 1) = 0$		
$\therefore \quad a = \frac{3}{2}$ or 1	1A+1A	
	6	
(b) (i) If the area of $OAPB = \frac{3}{2}$,		
$2a^3 - 4a^2 + 3a = \frac{3}{2}$		
$\therefore \quad 4a^3 - 8a^2 + 6a - 3 = 0$ ....(*)	1A	
(ii) Let $f(a) = 4a^3 - 8a^2 + 6a - 3$		
$f(1.2) < 0$ (= −0.408) and $f(1.3) > 0$ (= 0.068)	1	Correct signs only
$\therefore$ (*) has a root lying between 1.2 and 1.3		

Interval	Mid-value a_i	$f(a_i)$
$1.2 < a < 1.3$	1.25	− (−0.1875)
$1.25 < a < 1.3$	1.275	− (−0.0643)
$1.275 < a < 1.3$	1.2875 (1.288 etc)	+ (+0.0007)
$1.275 < a < 1.2875$	1.28125	− (−0.0321)
$1.28125 < a < 1.2875$	1.284375	− (−0.01578)
$1.284375 < a < 1.2875$	1.2859375	− (−0.00757)
$1.2859375 < a < 1.2875$		

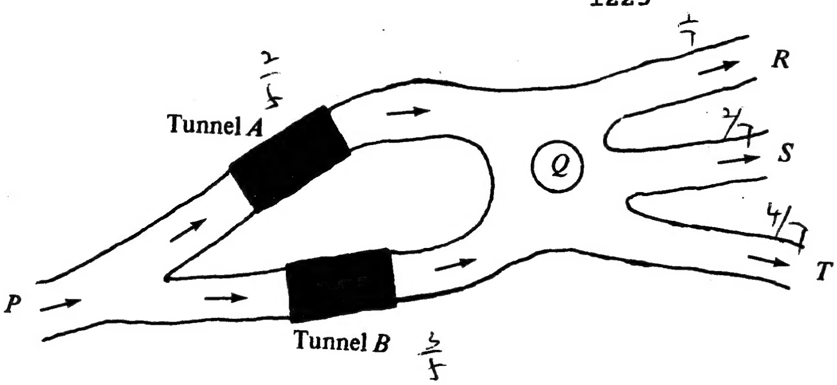
Marks	Remarks
1M+1A	1M for testing sign at mid-value
1M	Choosing correct interval

$\therefore \quad a = 1.29$ (corr. to 2 d.p.) — 1A — Check last interval, $a \approx 1.2874$

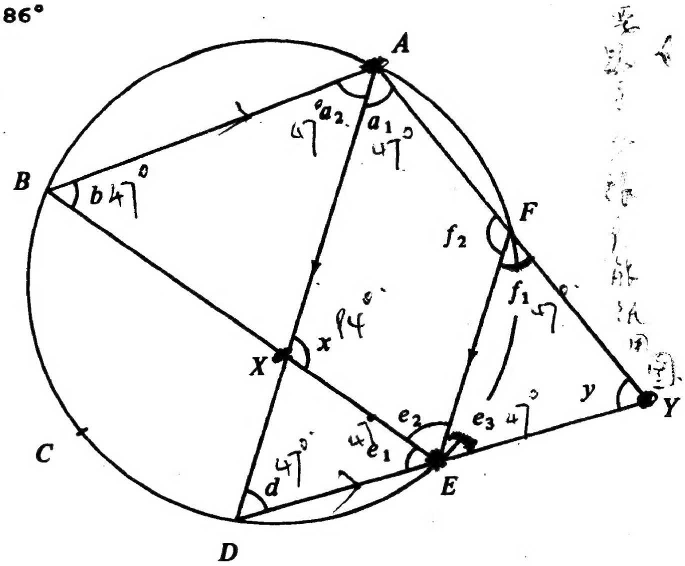
6



Solutions	Marks	Remarks
10. (a) The probabilities that a car leaving P will		
(i) pass through $B = 1 - \frac{2}{5} = \frac{3}{5}$ (= 0.6) (P_1)	1A	
(ii) not arrive at $T = 1 - \frac{4}{7} = \frac{3}{7}$ (= 0.429)	1A	$\frac{1}{7} + \frac{2}{7}$
(iii) arrive at R through Tunnel B $= \frac{3}{5} \times \frac{1}{7}$	1M	$P_1 \times \frac{1}{7}$
$= \frac{3}{35}$ (= 0.0857)	1A	
(iv) pass through Tunnel A but not arrive at R		
$= \frac{2}{5} \times (1 - \frac{1}{7})$	1A	$\frac{2}{5} \times \frac{2}{7} + \frac{2}{5} \times \frac{4}{7}$
$= \frac{12}{35}$ (= 0.343)	1A	
	6	
(b) (i) The probability that the first one will arrive at R and the second one at S		
$S = \frac{1}{7} \times \frac{2}{7} = \frac{2}{49}$ (= 0.0408) (P_2)	1A	Award 1A if $\frac{2}{49}$ given as answer
The probability that one of them will arrive at R and the other one at S		
$S = 2 \times \frac{1}{7} \times \frac{2}{7}$	1M	$P_2 \times 2$
$= \frac{4}{49}$ (= 0.0816)	1A	
(ii) The probability that both cars will arrive at S with the first one through Tunnel A and the second one through Tunnel B		
$= \frac{2}{5} \times \frac{2}{7} \times \frac{3}{5} \times \frac{2}{7} = \frac{24}{1225}$ (0.0196) (P_3)	1A	Award 1A if $\frac{24}{1225}$ given as answer
The required probability $= 2 \times \frac{24}{1225}$	1M	$P_3 \times 2$
$= \frac{48}{1225}$ (0.0392)	1A	
	6	



Solutions	Marks	Remarks
11. (a) **Proof :**		
$\angle f_1 = \boxed{\angle a_1}$ ($\angle DAY$, etc.) (Corr. $\angle$s, $AD // FE$.)	1A	Accept a_1, etc.
But $\boxed{\angle a_1 = \angle e_3}$ (Ext. $\angle$, cyclic quad.)	1A	
$\therefore\ \angle f_1 = \angle e_3$		
$\therefore\ EY = \boxed{FY}$ (Sides opp. equal $\angle$s)	1A	
i.e. $\triangle EFY$ is isosceles	3	
(b) **Proof :**		
$\overset{\frown}{BCD} = \overset{\frown}{AFE}$ (Given)		
$\therefore\ \angle a_2 = \boxed{\angle d}$ (Equal arcs subtend equal $\angle$s at circumference)	1A	
$\therefore\ BA // DE$ (Alt. $\angle$s equal)	1	
(c) **Proof :**		
$\angle a_1 = \boxed{\angle f_1}$ (Corr $\angle$s, $AD // FE$)	1A	
But $\boxed{\angle f_1} = \angle b$ (Ext. $\angle$, cyclic quad.)	1A	
and $\angle b = \angle e_1$ (Alt. $\angle$s, $BA // DE$)		
$\therefore\ \angle a_1 = \boxed{\angle e_1}$	1A	
$\therefore$ A , X , E , Y are concyclic. (Ext. $\angle$ equals int.opp. $\angle$)	3	
(d) **Solution :**		Note that
$\angle f_1 = 47°$	1A	$a_1 = a_2 = b = d = f_1$ $= e_1 = e_3$
$\angle y = 86°$	1M+1A	$y = 180° - f_1 - e_3$ or $y = 180° - x$
$\angle x = 94°$	1M+1A	$x = 180° - y$ or $x = b + a_2$ or $x = e_1 + d$
	5	



Solutions	Marks	Remarks
12. (a) (i) Capacity of funnel $= \frac{1}{3}\pi(9)^2 \times 20$	1A	
$= 540\pi$ cm^3	1A	
(ii) Vol. of water : total vol. of oil and water : cap of funnel		
$= 10^3 : 15^3 : 20^3$	1A+1A	1A for 10:15:20 or $10^2:15^2:20^2$
$= 2^3 : 3^3 : 4^3$ (= 8:27:64)	1A	
$\therefore$ vol. of water : vol. of oil : capacity of funnel		
= 8:19:64	1A 6	
(b) Let the depth of water be h cm.		
Capacity of bottom part $= \frac{2}{3}\pi \cdot 3^3$	1A	
$= 18\pi$ (cm^3)		
67.5π $540\pi \times \frac{8}{64} = \pi \times 3^2(h-3) + 18\pi$	1M	Equating vol. of water in two forms
$\therefore$ depth $= 8\frac{1}{2}$ cm	1A 3	
(c) Vol. of water : vol. of oil = 8:19		
$\therefore$ depth of water : depth of oil $= 2 : \sqrt[3]{19}$	2M	
$\therefore$ depth of oil $= 10 \times \frac{\sqrt[3]{19}}{2} = 5\sqrt[3]{19}$ cm (13.3 cm)	1A 3	

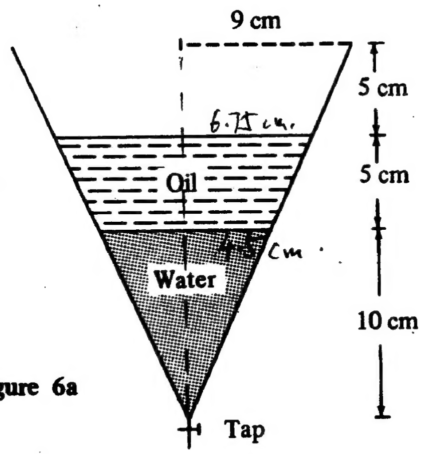


Figure 6a

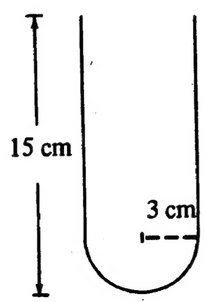


Figure 6b

Solutions	Marks	Remarks
Alternatively:		
$\frac{1}{3}\pi(4.5)^2(10)$		
12. (a) (ii) Vol. of water $= \frac{1}{3}\pi\ (9 \times \frac{10}{20})^2 \times 10\ (= 67.5\pi\ (\text{cm}^3))$	1A	
$\frac{1}{3}\pi(6.75)^2\ 15$		
Vol. of water + oil $= \frac{1}{3}\pi\ (9 \times \frac{15}{20})^2 \times 15\ (= 227.8125\pi\ (\text{cm}^3))$	1A	
$\therefore$ vol. of water : vol. of oil +water : cap. of funnel		
$= 67.5\pi : 227.8125\pi : 540\pi$		
$= 8 : 27 : 64$	1A 必定要寫出比	
Vol. of water : vol. of oil : cap. of funnel		
$= 8 : 19 : 64$	1A	
(c) Let the depth of the oil be h cm, the radius of the oil surface be r cm.		
Then $\frac{r}{h} = \frac{9}{20}$		
Volume of oil remaining $= \frac{1}{3}\pi r^2 h$		
$= \frac{1}{3}\pi(\frac{9h}{20})^2 h\ (\text{cm}^3)$	1M	Sub r
But volume of oil $= 540\pi \times \frac{19}{64}\ (\text{cm}^3)$		
160.3125π		
$\frac{2565}{16}\pi$ $\quad 540\pi \times \frac{19}{64} = \frac{1}{3}\pi(\frac{9h}{20})^2 h$	1M	
$\frac{135 \times 19}{16} = \frac{27}{400}h^3$		
Depth $= 5 \times \sqrt[3]{19}$ cm (13.34 cm).	1A	

$$\frac{540\pi(\frac{19}{64})}{640\pi} = \left(\frac{h}{20}\right)^3$$

$$\frac{135}{2}\pi : \frac{3645}{16}\pi : 540\pi$$

Solutions	Marks	Remarks
13. (a) $C = (9,7)$ (or $x = 9$, $y = 7$)	1A	5,7 (m...)
Radius $= \sqrt{9^2 + 7^2 - 105} = 5$	1A / 2	
(b) Putting $y = mx$,		
$x^2 + (mx)^2 - 18x - 14(mx) + 105 = 0$	1A	
$(1 + m^2)x^2 - (18 + 14m)x + 105 = 0$		
As x_1, x_2 are the roots, $x_1 x_2 = \dfrac{105}{1 + m^2}$	~~1A~~ / 2	Only awarded if above correct 毋須寫 x_1, x_2 are the roots.
(c) $OA = \sqrt{x_1^2 + y_1^2}$	1A	(optional).
$= \sqrt{x_1^2 + (mx_1)^2}$	1A	
$(= (\sqrt{1 + m^2})x_1)$		2 + 1
$OB = \sqrt{x_2^2 + y_2^2} = \sqrt{x_2^2 + (mx_2)^2} (= (\sqrt{1 + m^2})x_2)$	1A	
$\therefore \quad OA \times OB = (1 + m^2) x_1 x_2$		
$= 105$	1A / 4	
(d) Let M = mid-point of AB. If $CM = 3$,		
$AM = \sqrt{5^2 - 3^2}$ $(= 4)$	1M (半徑錯誤，但[illegible])	
$\therefore \quad AB = 2 \times 4 = 8$	1A	錯亦可
Let $OA = x$, then	$OA \times OB = 105$	**OR**
$x(x + 8) = 105$	1M	$OM = \sqrt{OC^2 - CM^2}$
$x^2 + 8x - 105 = 0$		$= \sqrt{9^2 + 7^2 - 3^2}$ 1M
$(x - 7)(x + 15) = 0$		$= 11$
$\therefore \quad x = 7$ (as $x \neq -15$)	1A / 4	$\therefore OA = OM - AM$ $= 11 - 4 = 7$ 1A

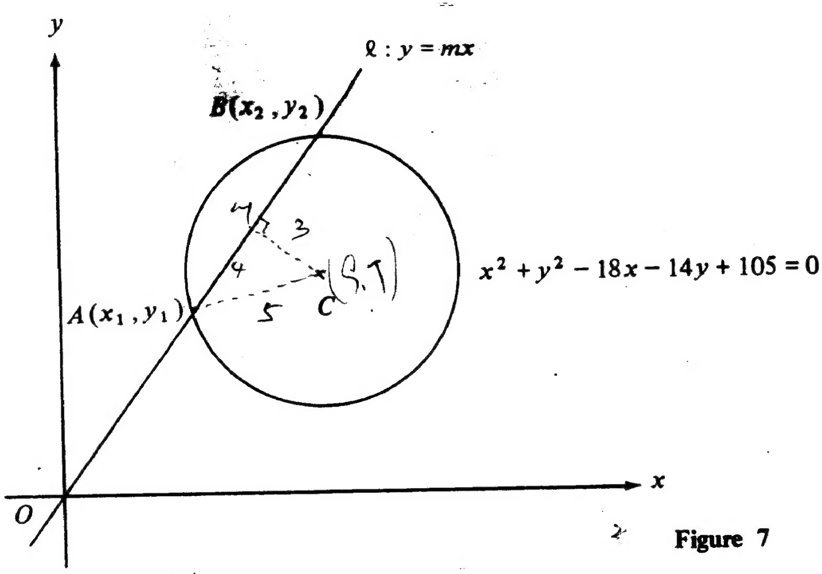
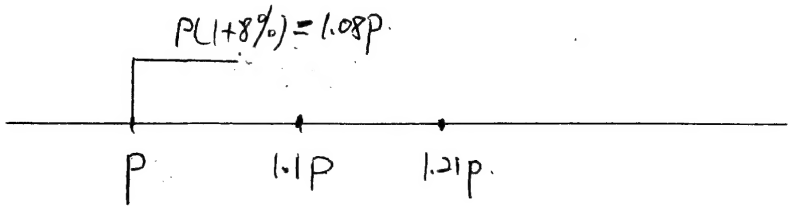


Figure 7

Solutions	Marks	Remarks
14. (a) The common ratio $= \dfrac{b}{a}$	1A	
The sum to n terms $= \dfrac{a^n\left[1 - (\frac{b}{a})^n\right]}{1 - \frac{b}{a}}$	1M	or $\dfrac{a^n - \frac{b}{a}(ab^{n-1})}{1 - \frac{b}{a}}$
$= \dfrac{a(a^n - b^n)}{a - b}$ $\left(= \dfrac{a^{n+1} - ab^n}{a - b}\right)$	1A 3	
(b) (i) The balance at the end of		
(1) the 1st year = $\$1.08P$	1A	$= (1 + 8\%)P$
(2) the 2nd year = $\$(1.08^2P + 1.1 \times 1.08P)$	1A+1A	$= 1.1664P + 1.188P = 2.3544P$
(3) the 3rd year = $\$(1.08^3P + 1.1 \times 1.08^2P + 1.1^2 \times 1.08P)$	1A	$= 3.849552P$
(ii) At the end of the nth year, the balance		
$= \$P[1.08^n + 1.08^{n-1} \times 1.1 + 1.08^{n-2} \times 1.1^2 + \cdots + 1.08^2 \times 1.1^{n-2} + 1.08 \times 1.1^{n-1}]$		
$= \$P\dfrac{1.08(1.08^n - 1.1^n)}{1.08 - 1.1}$	2A	
$= \$54P(1.1^n - 1.08^n)$	1 7	
(c) In n years' time, the flat is		
worth $\$1080000 \times 1.15^n$	1A	
Put $P = 20000$, the amount in the man's account		
$= \$1080000(1.1^n - 1.08^n)$		
$< \$1080000 \times 1.15^n$	1A 2	



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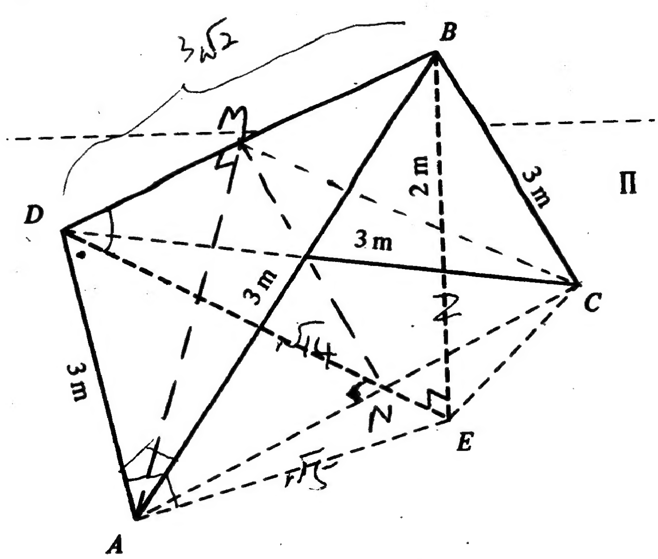
$(1.08P + 1.1P) \times (1 + 8\%)$

$= (1.08P + 1.1P) \times 1.08$

$= 1.08^2P + 1.1 \times 1.08 \times P$

Solutions	Marks	Remarks
15. (a) $BD = \sqrt{3^2+3^2} = 3\sqrt{2}$ m $(\sqrt{18}$ m$)$	1A	4.24, withhold 1 mk if answers not in surd form
$ED = \sqrt{BD^2 - BE^2}$		
$= \sqrt{18-4}$		
$= \sqrt{14}$ m	1A	3.74
$AE = \sqrt{BA^2 - BE^2}$		
$= \sqrt{9-4}$		
$= \sqrt{5}$ m	1A 3	2.24
(b)		
$\cos\angle ADE = \dfrac{3^2 + (\sqrt{14})^2 - (\sqrt{5})^2}{2\times 3\times\sqrt{14}}$ $(= 0.8018)$	1M+1A	
$\therefore$ $\angle ADE = 36.7°$	1A	36.5°~36.8°
Alternatively :		
As $\angle DAE = 90°$	1A	Follow through if omitted
$\tan\angle ADE = \dfrac{AE}{AD} = \dfrac{\sqrt{5}}{3}$	1M	
$\therefore$ $\angle ADE = 36.7°$	1A 3	36.5°~36.8°
(c) $\sin\angle BDE = \dfrac{2}{\sqrt{18}}$ $(= 0.4714)$	1M	or $\tan\angle BDE = \dfrac{2}{\sqrt{14}}$
$\therefore$ $\angle BDE = 28.1°$ (28.1255)	1A 2	or $\cos\angle BDE = \dfrac{\sqrt{14}}{\sqrt{18}}$



Solutions	Marks	Remarks
(d) Let M be a point on BD such that (or mid-pt of BD,etc.) $AM \perp BD$. We have $DM = MB$ as $AB = AD$.	1M	Considering AM
Let N be the mid-point of AC.		
Then $MN \perp AC$ as $AM = MC$.		
Similarly $DN \perp AC$.		
Now $\sin\angle ADE = \dfrac{AN}{AD}$		See also alt. solution
$\therefore \quad AN = 3\sin 36.7°$ m $(= 1.7928)$	1A	3 sin 36.5°~ 3 sin 36.8°
$AM = \dfrac{1}{2}BD = \dfrac{3}{2}\sqrt{2}$ m		
$\sin\angle AMN = \dfrac{AN}{AM} = \dfrac{3\sin 36.7°}{\frac{3}{2}\sqrt{2}}$ $(= 0.84515)$	1M	Attempt to find $\angle AMN$ or $\angle AMC$
$\therefore \quad \angle AMN = 57.69$ (57.6885)		
$\therefore \quad \angle AMC = 2 \times 57.69 = 115°$ $(\sim 116°)$	1A	
	4	

Alternatively:

Now $\sin\angle ADE = \dfrac{AN}{AD}$		
$\therefore \quad AC = 2AN = 2 \times 3\sin 36.7°$ m $(= 3.5858)$	1A	
$AM = \dfrac{1}{2}BD = \dfrac{3}{2}\sqrt{2}$ m		
By the cosine formula,		
$\cos\angle AMC = \dfrac{(\frac{3}{2}\sqrt{2})^2 + (\frac{3}{2}\sqrt{2})^2 - (2 \times 3\sin 36.7°)^2}{2(\frac{3}{2}\sqrt{2})(\frac{3}{2}\sqrt{2})}$	1M	Attempt to find $\angle AMC$
$= -0.4286$		
$\therefore \quad \angle AMC = 115°$ $(\sim 116°)$	1A	

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1. (a) $\frac{\pi}{6}$ (radian)

(b) $x = 150°$ ($\frac{5\pi}{6}$, 2.62)

(c) $\cos A$

2. (a) $p + q$

(b) -2

(c) $\sqrt{3} - \sqrt{2}$

3. (a) $y \geq \frac{1}{2}$

$2x - y \geq 2$

$3x + 5y \leq 30$

(b) 16

4. (a) (i) $x^2 - 2x = x(x - 2)$

(ii) $x^2 - 6x + 8 = (x - 2)(x - 4)$

(b) $\frac{1}{x^2 - 2x} + \frac{1}{x^2 - 6x + 8} = \frac{1}{x(x-2)} + \frac{1}{(x-2)(x-4)}$

$= \frac{(x-4) + x}{x(x-2)(x-4)}$

$= \frac{2x - 4}{x(x-2)(x-4)}$

$= \frac{2}{x(x-4)}$ $\left(= \frac{2}{x^2 - 4x} \right)$

5. (a) Slope of $L_2 = \frac{1}{2}$

Slope of $L_1 = -2$

Equation of L_1 : $y - 5 = -2(x - 10)$

i.e. $2x + y - 25 = 0$

(b) Solving $\begin{cases} x - 2y + 5 = 0 \\ 4x + 2y - 50 = 0 \end{cases}$

$5x - 45 = 0$

$x = 9$ (or $y = 7$)

$\therefore$ L_1 and L_2 meet at (9, 7)

6. For distinct real roots
$\Delta = (2k)^2 - 4(k + 6) > 0$

$4k^2 - 4k - 24 > 0$

$(k + 2)(k - 3) > 0$

$\therefore$ $k < -2$ or $k > 3$

7. (a) $\angle AOB = \frac{360°}{5} = 72°$ $\left(= \frac{2\pi}{5} = 1.26 \text{ radians} \right)$

Area of $\triangle OAB = \frac{1}{2}(10)(10)\sin 72°$

$= 47.6$ (47.5528)

(b) Area of sector $OAB = \frac{1}{5} \cdot \pi 10^2$

$= 20\pi$ (62.83)

Area of shaded part $= 20\pi - 47.55$

$= 15.3$ (15.2790)

8. (a) Total score of the team $= 70(m + n)$

(b) Total score is also equal to $75m + 62n$.

$75m + 62n = 70(m + n)$

$5m = 8n$

$m : n = 8 : 5$

(c) The number of men $= 39 \times \frac{8}{8 + 5}$

$= 24$

9. (a) (i) Area of $OAPB = a \times b$

$= a(2a^2 - 4a + 3)$

$= 2a^3 - 4a^2 + 3a$

(ii) For $OAPB$ to be a square, $a = b$

$a = 2a^2 - 4a + 3$

$2a^2 - 5a + 3 = 0$

$(2a - 3)(a - 1) = 0$

$\therefore$ $a = \frac{3}{2}$ or 1

(b) (i) If the area of $OAPB = \frac{3}{2}$,

$2a^3 - 4a^2 + 3a = \frac{3}{2}$

$\therefore$ $4a^3 - 8a^2 + 6a - 3 = 0$..........(*)

(ii) Let $f(a) = 4a^3 - 8a^2 + 6a - 3$

$f(1.2) < 0$ ($= -0.408$) and $f(1.3) > 0$ ($= 0.068$)

$\therefore$ (*) has a root lying between 1.2 and 1.3

92-1